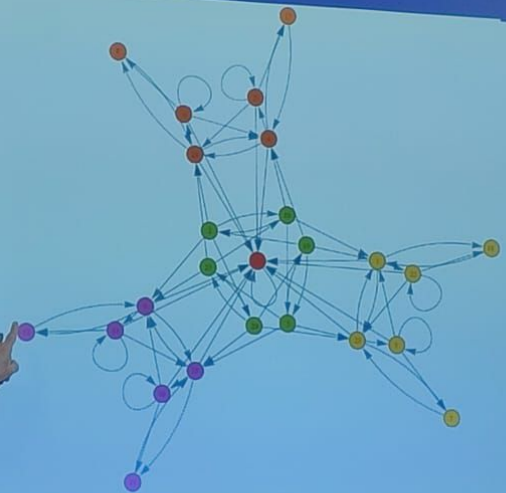






### The associated Markov chain



$$11 = f_6 + f_4$$

$$v(11) = 1 \quad x = (2, 0)$$

$$v(f_6 + f_4) = 1$$

$$v(f_{6 \rightarrow 4} + f_{4 \rightarrow 6}) = 1$$

	$f_6$	$f_5$	$f_4$	$f_3$	$f_2$
$n=11$	8	5	3	2	1
+	0	1	0	0	0
+	0	1	1	1	1
-	1	0	0	1	1

$$11 = f_6 + f_4$$

$$= f_5 + f_4 + f_3 + f_2$$

$$= f_6 + f_3 + f_1$$

Canonical/Zeckendorf representation

Every positive integer  $n$  can be written uniquely as

$$n = f_{k_1} + f_{k_2} + \dots + f_{k_r}$$

where  $r$  depends on  $n$  and

$$k_1 < k_2 < \dots < k_r \leq k_{r-1} - 1$$

Set

$$f_k = \alpha_k f_{k-1} + f_{k-2} \quad (k \geq 2)$$

$$f_0 = 0, f_1 = 1$$

$$\prod_{k=1}^{\infty} (1 - x^{f_k}) = \sum_{n=0}^{\infty} (-1)^n x^n$$


on

$$B_i = \begin{bmatrix} \lfloor \frac{x_i - x_{i+1}}{2} \rfloor & \lfloor \frac{x_i - x_{i+1} - 1}{2} \rfloor \\ -1 & -1 \end{bmatrix}$$

$x_2, \dots, x_r$  with  $r \geq 2$  and  $x_i \in \{0, 1, 2, 3\}$ , set  $M(x, x_r) = B_2 B_3 \dots B_{r-1}$ .

$(x, x_r)$  is given by the vector-matrix-vector

$$(x_2 - 2), \nu(x_1 - x_2 - 3) M(x, x_r) \begin{bmatrix} 1 - \lfloor \frac{x_1}{2} \rfloor \\ -1 \end{bmatrix}$$

representation  $n = f_{k_1} + f_{k_2} + \dots + f_{k_r}$  with  $n \equiv 3 \pmod{4}$ , then  $v(n) = 0$ .

Handwritten notes on a whiteboard:

	$f_6$	$f_5$	$f_4$	$f_3$	$f_2$
	5	4	3	2	1
+	0	1	0	0	0
+	0	1	0	0	0
-	1	0	0	0	0

$n=11$

